LOOKING THROUGH ROCKS

F. Chayes *Carnegie Institution of Washington*

AIMS AND PURPOSES

Thin enough sections of all crystalline rocks are transparent. They are also among the most beautiful objects studied under the microscope. Most of them consist of aggregates of minerals that can be distinguished from each other by striking differences in optical properties. The science of petrography is devoted to the study of such aggregates.

Although several thousand mineral species are now known, fewer than 15 probably comprise more than 95% of the earth's rocks, or what is called the *lithosphere*. Further, only seven or eight are likely to occur in appreciable amounts in any one rock. Hence, information about the amounts of different minerals it contains usually plays a key role both in the description and classification of a rock and in attempts to decipher its geochemical history.

An estimate of the amounts of the different minerals actually present in a rock is called a *modal analysis*, or *mode*. Although the petrographic use

of this once standard philosophical term goes back only to the beginning of our century, what are now called modal analyses began appearing in the technical literature just before the 1850s.

When the geologist uses the term "rock" he is usually thinking of a large mass, or volume of material, underlying an area measurable at least in acres and often in square miles. He can bring home only a few small pieces, or *hand specimens,* of any particular rock, however. The selection of these is the first in a long series of sampling operations that intervene between rocks and scientific knowledge about them.

In the case of modal analysis, for instance, the process consists, in principle, of identifying and estimating the volume of each of a large number of mineral grains, summing the volumes of grains of each kind, and dividing each of these sums by the total measured volume. No one would attempt such a measurement on an entire rock; even the small hand specimen the geologist brings home to his laboratory may contain millions of grains! It is always impractical and usually impossible to separate well consolidated rocks into single grains that can be measured individually, and it is clearly impossible to measure grain volumes directly unless this can be done. So, after all, we do not measure grain volume directly; rather, we measure something statistically related to grain volume, namely, the *area* the section of the grain occupies in a random plane intersecting it. This plane may be a flat surface ground onto a slab or hand specimen of the rock. It is more likely to be the surface of a microscopic preparation called a *thin section,* for many of the mineral identifications required in modern petrography can only be made under the optical microscope or one of its more powerful sisters, the electron microscope and the electron probe.

Our objective is knowledge of the volume composition of a rock body measurable in cubic meters or kilometers. We reach this knowledge by sampling minute amounts of a volume-related characteristic, random cross-sectional area. In the successive stages of sampling, we continually reduce the sample volume, from mountain or hillside to outcrop to hand specimen. In the final stage, volume disappears entirely and its place is taken by area. We shall be concerned here with the product of this final sampling, with the ways of measuring relative areas, and with the reasons we have for supposing that measurements of relative areas contain reliable information about relative volumes.

THE THREE MAJOR FORMS OF MODAL ANALYSIS

In modal analysis we partition a reference area into a set of subareas by direct areal measurement, by measurement of line segments cutting each type of subarea, or by counting points that fall in them. Each technique must satisfy substantially the same basic requirements. We must show that it yields estimates of the relative areas of essential minerals in a plane section of the rock. Then we must show that these relative areas are sound estimates of relative volumes.

If a plane reference surface is sawed and ground on the face of a rock, the measurements will be made at low magnification in reflected light. If, on the other hand, the reference area is the upper surface of a thin section, it will be viewed at considerably higher magnification and in transmitted light. In either case, the result of measurement will be a set of numbers thought to be proportional to the areas occupied by the essential minerals of the rock.

Such numbers were once obtained by tracing the outlines of the various mineral sections onto translucent paper and hence onto tinfoil or cardboard, cutting along the traces of grain or section margins, pooling those of each species, and weighing the pooled fractions. This is the way modes were first obtained, in 1848, by the French mining engineer A. Delesse. Delesse worked on tracings made from rock slabs, but in 1856, Sorby, the great English microscopist, treated tracings of much enlarged projections of microscopic objects in the same way. In 1859, in the *Origin of Species,* Darwin recorded making similar measurements on "Professor H. D. Rogers's beautiful map of Canada," thus discovering that in that country areas of "metamorphic . . . and granitic rocks exceed, in the proportion of 19 to 12.5, the whole of the newer Paleozoic formations." Neither Sorby nor Darwin says a word about Delesse, but it is possible that neither knew of his work.

It is intuitively obvious that if the cardboard or tinfoil is of uniform gauge and the cutting and weighing are exact, the numbers yielded by the Delesse method will indeed be proportional to mineral areas. The "theoretical" basis of the cutout procedure is beyond reproach, but no one has ever attempted a systematic study of the accuracy or precision of the results. In fact, almost no one has used the method for routine modal analysis. Shortly after it was proposed, petrographic interest shifted from the study of hand specimens to the study of microscopic, or *thin,* sections.

Sorby's tracings of enlarged microscopic reference areas were soon substituted for tracings made from polished slabs, but the area of rock represented by one such drawing was a very small portion of the usable area of even a mediocre thin section. Making and measuring a single drawing was a time-consuming chore, and when a prominent petrographer suggested that reliable estimates of the modal composition of a single thin section of a rather common rock would require preparation, tracing, and cutting up of drawings from projections of at least a couple of dozen microscopic fields, the profession seems to have decided to wait for better methods. Attempts to replace the cutting and weighing step by direct areal measurements continued for some time but never attained any real popularity.

The Delesse method cannot be said to have failed; it simply has not been used. But its day may be approaching. The scanning of an image by a TV tube is, for all practical purposes, an area measurement of the Delesse type. It is also virtually instantaneous, and the TV tube by 1971 was beginning to be used for measuring relative areas of materials reflecting (or transmitting) light with different intensities, though the procedure was still far from routine.

Toward the close of the last century direct areal measurement was replaced by linear measurement. The Viennese geologist A. Rosiwal, who was responsible for this change, worked at first on rock slabs, which he ground flat and polished. Inscribing lines on the flat surface, he measured the lengths of segments lying in each of the minerals exposed in it. The proportion of a line lying in a particular mineral species was his estimate of the proportion of that mineral in the rock. It is interesting and rather startling that the lines along which Rosiwal first made his measurements were neither straight nor parallel. His drawings clearly show that the *measurement lines,* or *rock threads,* he inscribed on the surface of a slab were strongly curved and intersected in a complicated, unsystematic fashion.

The petrographic microscope, which had been a novelty when Sorby made his drawings, was standard equipment in Rosiwal's day. He soon attempted to exploit the vastly improved mineral identification made possible by it, and the inadvertent approach to random sampling suggested by his curved and intersecting *rock threads* was ended. Under the microscope it is difficult to use reference lines that are anything but straight, and it is by far simplest to use straight lines that are parallel to each other. *Rosiwal analysis* now refers exclusively to the measurement of mineral intercepts along a set of parallel straight lines. In the original Rosiwal procedure these lines were rulings in the eyepiece of a microscope. In modern equipment, rotation of the calibrated screw of a mechanical stage in which the specimen is clamped moves it past a reference point, usually the intersection of the cross hairs in the microscope eyepiece. Although the measurement technique proposed by Rosiwal was hopelessly time-consuming, the measurement itself seemed so proper on the basis of geometric intuition that he did not bother to justify it. The first attempt at formal justification did not appear until 1913. Few persons applied the Rosiwal technique in routine work, for by his method the analysis of a single thin-section area of less than a square inch required a number of hours. Subsequent mechanical improvements finally reduced the time requirement substantially, and the method was just beginning to find wide application when the whole basis of measurement shifted once more.

Just as it had previously been discovered that it is much easier to measure lines than areas, it was now discovered that it is much easier to count points than to measure lines, as the Russian mineralogist A. A. Glagolev suggested in 1933. Like most Russian work of that period, this suggestion was ignored in the West, where the advantages of point counting were discovered independently 16 years later. A short English note on the subject by Glagolev in an American mining journal seems to have escaped almost unnoticed, probably for the reason that the journal was not one widely read by either petrographers or metallurgists. Except for elaboration of the instrumentation, the principal effect of which seems to have been to increase the cost of the operation, the replacement of line measurement by point counting brings to a close the preelectronic development of the art of point counting.

ERRORS IN MEASURING AND IMPRECISION IN THINKING

The analogy between the areal method of Delesse and the linear method of Rosiwal is evident. As the distance between lines decreases, the precision of estimates of area yielded by the latter increases. In the limit, as the traverse interval becomes infinitesimal. the distinction between a Rosiwal analysis and an (errorless) Delesse analysis vanishes. For what now seems a rather bizarre reason, no systematic comparison of the precision and accuracy of the two procedures seems to have been made.

The reason is just that until long after the demise of direct areal measurement, traverses were, in fact, neither uniformly nor randomly spaced over the reference area in Rosiwal analysis. Rosiwal always regarded his *rock thread*—the line along which measurements were to be made—as a sample of the rock rather than of the surface on which it was drawn or imagined. When he abandoned the curvilinear, intersecting "threads" of his earlier work in favor of straight, parallel traverses, he prescribed only that the distance between traverses should be adjusted to insure that no grain was cut by more than one traverse. Before 1923, no one suggested in print that a uniform traverse interval ought to be used in any particular analysis or set of analyses. Even now, the supposed undesirability of traversing any grain more than once is rediscovered every few years. Failure to insist on a traverse interval that either varied randomly or was uniform—a failure that makes it difficult if not impossible to evaluate precision and accuracy—stems from something very like what is nowadays called the confusion of target and sample populations. The target is the rock; the sample is the surface of the thin section. The ultimate aim may be knowledge of the composition of a hand specimen, an outcrop, a hillside, or a continent. But the sample always intervenes between the observer and the target, and if we wish to reach useful inferences about the target our first business is to obtain reliable statistics from the sample. In an individual Rosiwal, Delesse, or other modal analysis, we are not directly estimating the composition of a rock; rather, we are estimating the proportion of a reference area occupied by each of the essential minerals.

How well can we do this? However unsatisfactory the situation as regards the earlier techniques, the theoretical precision, or reproducibility, of modal

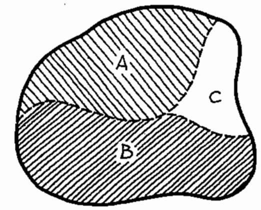

FIGURE 1
Schematic drawing of reference area showing regions occupied by each of three minerals. The relative areas of regions A, B, and C can be estimated by point counting

analysis by point counting is fairly well understood. Consider the heavy line of Figure 1 as the boundary of a reference area, whether thin section or polished slab, and the blank, stippled, and striped regions within the boundary as areas occupied by each of three minerals. Suppose that we select points within the boundary of the reference area and record the subfield, or mineral, in which each point falls. If the points are chosen simply at random in every sampling, the probability that any point selected will lie in, say, area A, is precisely the ratio of area A to the reference area, the very thing we want to know. If we select N points at random, the number falling in A should be approximately $N_a = Np_a$, where p_a is the (unknown) proportion of the reference area occupied by A. In practice, we use the observed ratio, N_a/N, as an estimate of p_a. Now N_a/N will nearly always differ somewhat from p_a, but the error involved in estimating p_a in this fashion is well known and can again be estimated directly from N and N_a.[1] Exactly the same situation holds for minerals B and C, and however many more there may be. Estimating areal proportions by selecting points simply at random within a reference area is one way of generating what some readers of this book will recall from high school mathematics as the binomial distribution.

We may draw one valuable lesson from this picture of the point-counting process even though, as we shall see, it does not apply exactly to our problem. Clearly it does not matter whether mineral A occurs in one large continuous subarea or is scattered over the reference area in a large number of discrete, isolated subareas. In a properly random sampling the probability that a point will fall on mineral A depends *only* on the proportion of the reference area that is occupied by A.

As with Rosiwal's technique, however, and for the same reason, the selection process actually used is systematic, not random; this is why we said the precision or reproducibility theory of modal analysis by point-counting is fairly well—instead of exactly—understood. Nowadays we certainly could

[1] For instance, if p_a is really 0.2 and $N = 100$, the observed values of N_a/N will almost all lie between 0.1 and 0.3, and most of them will be much closer to 0.2 than to 0.1 or 0.3.

arrange a suitably random scheme of sampling, but the convenient way to collect the data from a single reference area is to locate the points systematically, at regular intervals along uniformly spaced traverses. In fact, we use a rectangular point grid so the selection of points is systematic, not simply random. Once the position of the grid is chosen, the location of all the points is fixed in principle and varies in practice only because of unavoidable, but variable, slippage in the traversing mechanism of the mechanical stage. Although a priori evaluation of the random error of counts made on a rectangular grid is difficult, this error is readily estimated experimentally. A rather sizable body of data now indicates that the reproducibility of modal analysis based on points distributed on a rectangular grid is not substantially different from that associated with the same number of points distributed simply at random over the same area. (This, in fact, is why we discussed random point-counting error in such detail. As is often the case in the natural sciences, much may be learned from a simple statistical model even though the model is only approximate and may ultimately be modified or abandoned.) There are limitations, to be sure (for instance, the relation of distance between traverses to distance between points along a traverse must be such as to insure good coverage of the reference area by the systematic count), but for all practical purposes, the theoretical reproducibility of means determined by simple random counts of size N provides satisfactory estimates of the observed reproducibility of modal analyses based on N points distributed over a rectangular grid.

FROM AREAS TO VOLUMES

The sample statistics are thus in good order. We have a convenient way of estimating the "composition" of our reference areas, or thin sections, and we also have satisfactory knowledge and control of the precision of these estimates. Now we must examine briefly the relation between sample and target, the reason for supposing that measurements of areal proportions are sound estimates of volumetric proportions, so that observations on thin sections can tell us about the composition of rocks.

Delesse was acutely aware that it was a big step from areal to volumetric proportions, and seemed to sense it was a step he could not take. Lacking the modern vocabulary of sampling and estimation, which makes possible the discussion, and sometimes the solution, of such problems, he was obliged to rely heavily on geometrical intuition. In the end, he did not get much past the simple announcement that it seemed reasonable to suppose the proportion of the measurement area occupied by a mineral was the same as the volume percentage of that mineral in the rock from which the measurement areas had been obtained.

No one objected—indeed, the question was not raised again in print for nearly a century—but in retrospect, it seems quite possible that doubts about the validity of areal measurements as estimates of volumetric proportions were responsible for the sluggish development of the subject. The information was of a sort that every petrologist badly wanted. The period was one of considerable advancement in the instrumentation of other forms of microscopic observation. It is true that the instrumentation proposed by Delesse, Sorby, and Rosiwal was crude and inefficient, making the process of modal analysis painfully slow and difficult. But mechanical devices which vastly reduced the time requirements were in fact developed in the first quarter of the present century, and there was nothing about these instruments that would have taxed the ingenuity of the machinists and gadgeteers associated with the remarkable development of instrumentation for the sister sciences of optical mineralogy and crystallography in the last half of the nineteenth century. Had there been a demand for suitable measuring devices, it almost certainly would have been met. But there was no point in measuring relative areas unless they were sound estimators of relative volumes, and there was no assurance that this was so.

The situation today is very different, largely because of the development of a vocabulary specifically designed for analysis of problems such as this one. Basically, we are concerned with the properties of relative areas as estimators of relative volumes. It turns out that relative area is always a *consistent* estimator of relative volume, in the sense that the average of results for an increasingly large number of randomly chosen reference areas is less and less likely to differ from the true relative volume by any given amount, however small.

This is good, but it is not good enough. No one is going to measure an infinite or even a very large number of reference areas of the same sample volume. What we would like to be told is that relative area is a good estimator of relative volume, in the sense that the average *relative* area obtained from any number, however small, of reference areas is likely to be close to the true *relative* volume. Now this, unfortunately, is untrue. It is stretching things very little to say that during the century following Delesse's discovery petrologists refrained from exploiting modal analysis because they suspected some such defect in the area-volume relation, even though for most of that time language which would permit a concise description of their suspicions was either not available to or not known by them.

The remedy, however, is surprisingly simple. Areal proportions measured on sections parallel to any face of a parallelopiped do give good estimates of volumetric proportions in the parallelopiped. If all reference areas used in a particular study are of the same size and shape, the relative areas do average close to the relative volumes. As Delesse suspected, but could not prove, the possibility that this area-volume relation fails is by no means an insuperable obstacle. Instead, it is merely an altogether unexpected but sound reason for using, in any particular study, reference areas which are parallelograms of the same size and shape.

Given experimental evidence that the error of point counting follows a simple, well-known statistical rule and the a priori assurance that relative areal proportions may indeed be good estimators of volumetric proportions, we ought to be prepared to evaluate systematically the errors attaching to the remaining sampling steps required if, as Sorby insisted more than a century ago, "the mountains must indeed be examined under the microscope." There has been considerable progress in this direction, largely stimulated by the substitution of point counting for linear analysis in the decade following World War II.

But it is one of the rules of natural science that a new or improved analytical technique creates a demand for far more data than can be provided. The modal analyst of 1970 can do in 15 minutes what his scientific forebearers of 1920 probably could not do in less than two hours. This striking improvement has created interest in sampling problems whose successful solutions require far more than eight times the number of analyses that the same amount of work would have generated in 1920. The petrologist works with a number of closely interrelated sets of variables characterizing rock composition, and if he cannot get enough information from one set, he turns to another. So interest has shifted recently to rapid methods of chemical analysis, and after a flurry of productive activity extending from 1945 to about 1960, modal analysis seems to be caught in another of the standstills that have characterized its history. Probably the next great revival of interest in it will be prompted by successful electronic automation of the analytical process, as suggested above.

This brief review of what is obviously a highly specialized scientific activity may perhaps best conclude with a reminder that science itself is something more than a collection of scientific specializations (or, as they often seem, overspecializations). The practical day-to-day activity of all natural science does consist, for the most part, of learning more and more about less and less. It often happens, however, that the means by which we seek knowledge in one field are independently developed by, find application in, or are borrowed from, another. The potential yield of a forest, for instance, may be estimated by a kind of traverse sampling not unlike that used on microscope slides in petrographic point counting. What petrographers call modal analysis is probably more widely used in the study of metals and alloys than in the study of rocks. Application of the electron microscope to biological tissues has revealed an enormous amount of detail in the previously featureless cytoplasm, or nonnuclear portion, of the cell; here, too, modal analysis is beginning to find application. The objects being examined and the methods of observation differ widely from field to field, so widely, in fact, that a specialist in any one usually knows little or nothing about the others. But the *statistical*

concepts on which modal analysis rests are the same whatever the nature of the material being analyzed. A derivation of the area-volume relation developed for one field is applicable to all, as are the results of a properly designed study of precision or reproducibility. It has been said that although the nouns of the technical languages of various fields of specialized scientific activity are very different, the verbs are pretty much the same. To this we may add, extending the analogy a bit, that the roots of many of the verbs common to the diverse languages of the natural sciences are basically statistical.